# NMR Imaging of Fractal Fingering in Hele-Shaw Cells

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Viscous fingering is a well documented phenomenon (Saffmann and Taylor, 1958) which has generally been visualized through the medium of photography. Recent studies (Davies et al., 1992) have illustrated the use of nuclear magnetic resonance (NMR) imaging to visualize this phenomenon. The fingering produced by displacement of non-Newtonian fluids has been shown to be fractal (Nittmann et al., 1985), and this work shows that NMR imaging can be used both to visualize and quantitate the fractal nature of these fingers. NMR imaging has an advantage over photography to the extent that it can be used to visualize systems in nontransparent vessels and interactions between opaque liquids.

## NMR Data Acquisition Techniques

All data were acquired using an Oxford Research Systems Biospec 1 console, controlled by a Bruker Aspect 3000 computer, interfaced with an Oxford Instruments 2T, 31-cm horizontal bore, superconducting magnet with 20-cm purpose-built Helmholtz-Golay gradient coils. A sine-spaced birdcage radiofrequency probe (Bolinger et al., 1989) was used. All data processing, except where stated otherwise, was performed on a Sun4/150 workstation with a TAAC accelerator board using in-house software.

The image of the cell was obtained using a prefocused gradient recalled echo imaging sequence (Roberts et al., 1990) (Figure 1) which allows fast 2-D images to be obtained, which display a $T_1$-dependent signal intensity. Such prefocused pulses excite longitudinal components of the magnetization while "spoiling" or dephasing existing transverse components. As a result, the peak amplitude of the gradient-recalled echo formed after each pulse depends on the proportion of the excited magnetization, which has undergone longitudinal relaxation in the interpulse interval (TR). In contrast, conventional FLASH imaging (Haase et al., 1986) requires additional spoiling mechanisms to dephase transverse magnetization and prevent it fròm giving rise to spurious echos which interfere with the desired echoes and degrade the image contrast. Consequently, if TR (and so the overall imaging time) is kept short, species with short longitudinal relaxation times will appear relatively bright, while species with longer $T_1$'s will appear dimmer. The signal intensity from a gradient-recalled echo formed a time (TG) after the pulse application is given by the equation:

$$S\alpha\rho(1-e^{-TR/T_1})e^{-TG/T_2^*}$$

TG is therefore kept as short as possible ($\sim 1$ ms) to minimize the effect of $T_2^*$ on the image contrast. This condition is easily satisfied in this case, since the aqueous carboxymethylcellulose had a line width of $\sim 30$ Hz and therefore a $T_2^*$ corresponding to $\sim 10$ ms and so, relaxation during the gradient echo time may be neglected.

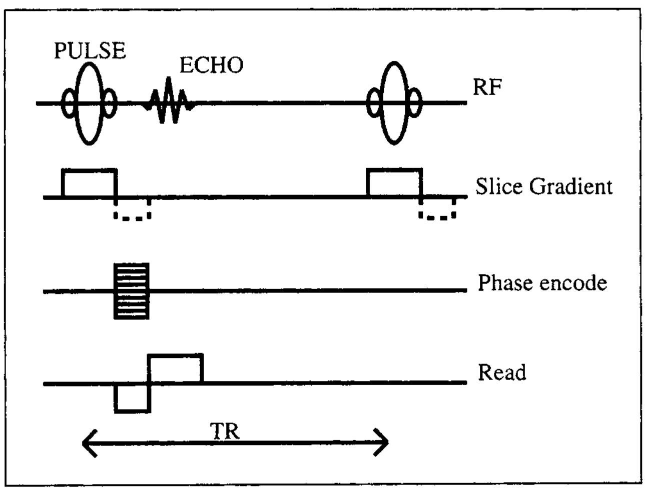


**Figure 1. Fast gradient recalled echo imaging sequence.**
The dotted line indicates the refocussing gradients necessary in conventional FLASH imaging.

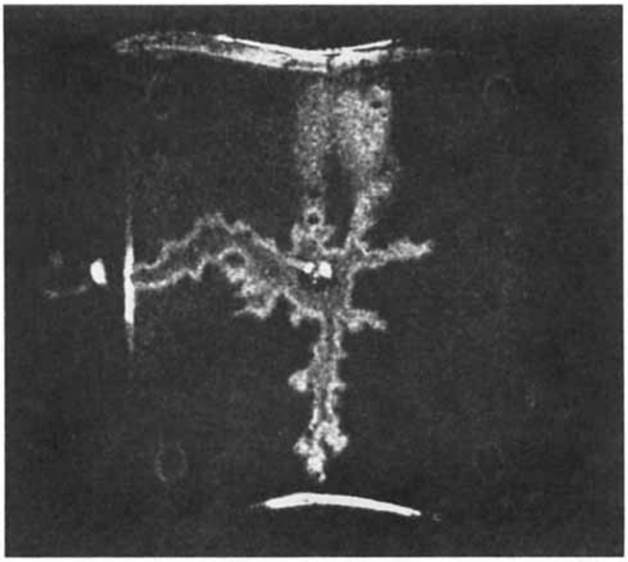

**Figure 2. Fingering pattern produced using a prefocused gradient recalled echo sequence.**

TE = 3.2 ms, TR = 103 ms, field of view = 7.14 cm, pixel resolution = 139 μm.

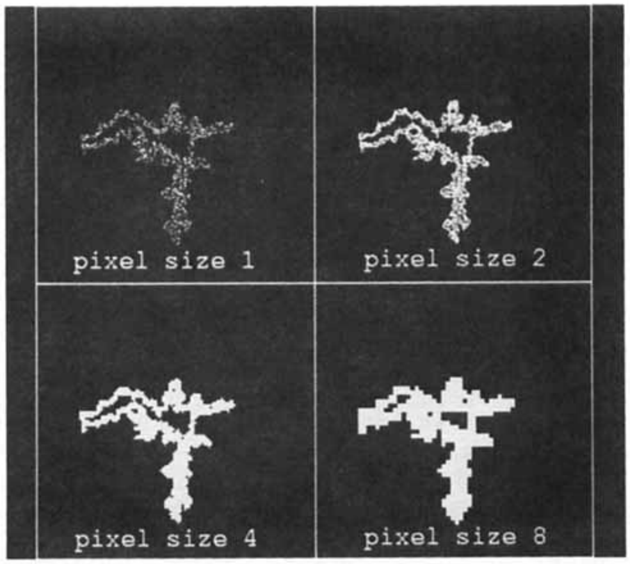


**Figure 3. Detection of edge using mask filtering routine.**

The four different images show the edge at effective pixel resolutions of 1, 2, 4 and 8.

The experiment was carried out using a repeat time of 103 ms and an echo time of 3.2 ms. Thus, the overall time to acquire an image with a resolution of 512 × 512 pixels was 53 seconds. In practice, the imaging time has to be kept as short as possible due to the short duration of the fingering pattern; but, the higher the resolution required, the longer the time needed. Therefore, a compromise has to be achieved between the time taken to acquired the image and the resolution attained. The pattern imaged in Figure 2 was obtained using six signal averages, giving an overall imaging time of 5.3 minutes. The pattern appears to remain stable over this time period.

Fractal fingers were produced using a radial Hele-Shaw cell, 5 $cm^2$ (Hele-Shaw, 1898), with an internal separation of ~1 mm. The pattern was produced by the displacement of 10% w/w Carboxymethylcellulose (CMC)-water paste by water doped with Manganese Chloride (2 mM $MnCl_2$), as shown in Figure 2.

## Data Processing

Calculation of the fractal dimension of the pattern produced in the Hele-Shaw cell can be achieved quickly and routinely on the image generated by the above sequence, using the method of "Mosaic Amalgamation" (Russ, 1977). A filter was passed over the image to detect the changes in intensity so that the edge of the pattern could be defined. The number of pixels defining the edge was then counted and an estimate of the length of the perimeter of the pattern obtained. The image was then "degraded" by averaging the number of pixels, and the filtering and counting processes repeated (Figure 3). A log-log plot of the length of the perimeter against the effective pixel size was constructed and the slope of the line calculated (Figure 4). The fractal dimension of the pattern ($D$) is given by:

$$D = 1 + |\text{slope}|$$

In practice, the edge of the fractal pattern is detected using a point detection filter mask (Gonzalez and Wintz, 1977). A 3×3 mask was scanned across the image, so that all but the very edge pixels were covered, and the sum of the products placed in the central square. This mask cancels any uniform areas and detects any large changes in pixel intensity, such as those that occur at the edge of the pattern.

The bright line at the edge of the fractal pattern is derived from the interface between the CMC (dissolved in distilled water) and the manganese chloride-doped water. This contrast must be due to a $T_2$ effect, since the $T_1$'s of the doped water and "doped CMC" effectively present at the interface are similar, 61 ms and 72 ms, respectively. The effect can only be tentatively attributed to a $T_2$ effect, because accurate meas-

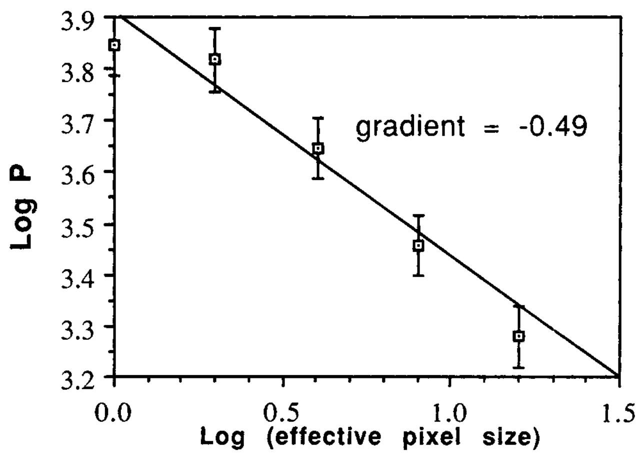


**Figure 4. Log-log plot of the estimate of the length of the perimeter of the sample against the effective pixel size.**

The fractal dimension, calculated from the slope, is 1.5.

urements cannot be made due to the differences between a phantom and the real system. This bright line around the edge of the pattern does, however, provide us with an unambiguous edge to the pattern, which the filtering process will detect easily.

## Results

From the image processing described above we have derived a fractal dimension of 1.5 for the pattern produced by the displacement of CMC by doped water. This compares favorably with the value of 1.4 quoted for the displacement of a polysaccharide with water by Nittmann et al. (1985).

## Conclusions

It has been shown that NMR imaging is a useful tool for the visualization of fractal fingering in 2-D systems. NMR imaging can also be used to visualize such systems in three dimensions and can also be used to distinguish between chemical species, such as water and hydrocarbons (Horsefield et al., 1990). It is in these areas that the real advantages of NMR imaging over optical methods lie.

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